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FOR M/G/1 MODELS

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This paper addresses two problems of interest in service system analysis: (a) that of making statistical, data-driven estimates of the long-run probability of a long delay, and (b) the assessment of rate of approach to a long-run system performance measure such as expected delay, the rate being characterized by a simple exponential, at least initially. Both are illustrated by reference to M/G/1 and related systems.

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ON INFERENCE AND TRANSIENT RESPONSE FOR M/G/1 MODELS

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This paper addresses two problems of interest in service system analysis: (a) that of making statistical, data-driven estimates of the long-run probability of a long delay, and (b) the assessment of rate of approach to a long-run system performance measure such as expected delay, the rate being characterized by a simple exponential, at least initially. Both are illustrated by reference to M/G/1 and related systems.

1. Introduction

The application of probability theory to a wide variety of congestion problems arising in communication systems has been well catalogued in many papers; the treatises by Borovkov (1984), Cohen (1969), Cooper (1972), Cox and Smith (1961), Feller (1966), Kleinrock (1975), Gross and Harris (1974), and many others attest to the attractiveness of the area for probability modelling. In this body of work, several features are noticeable. First, most of the elegant solutions obtained are in somewhat implicit form, being presented as functional equations, or, frequently, as integral (Laplace) transforms, generating functions, and sometimes as combinations of the above. Moments (particularly the first) of delays or number in queue under long-run conditions when steady state prevails are the most explicit and assessible performance summaries. Second, the results obtained are presented in terms of component distribution functions and stochastic processes (renewal, Poisson, etc.) that are taken as known; only rarely are issues addressed that arise when actual data is to be used as a basis for inference from the models; however, see Cox (1965). Third, only fragmentary comprehensible information concerning transient behavior, not to mention time-dependence of parameters is available; recently however work by Roth and Odoni (1983), Roth (1981), Abate and Whitt (1985), Lee (1985) has elucidated the simple exponential-like approach to the steady state displayed by a great many stochastic systems with stationary, time-homogeneous arrivals and service processes.

Instead of directly dealing with the above classes of problems much attention has been concentrated on modelling large networks of servers, particularly steady-state Markovian, cf. Kelly (1979) for an elegant treatment; see package programs created by AT&T (the D. Mitra PANACEA) and by IBM (work by Lavenberg, et al). Other numerical work by Neuts (1981) and associates has pioneered the exploitation of special structure of certain stochastic systems. And a considerable effort to create approximations, ranging from heavy-traffic and diffusion to, lately, WKB approaches, see Knessl, Matkowsky et al (1986) has been evident; see in particular Newell (1982) for pioneering work.

This paper deals specifically with approaches to the two classes of somewhat neglected problems referred to earlier: those of data-driven inference, and of assessing transient behavior. The approaches proposed are illustrated concretely

in terms of the simple M/G/1 system, but apply more widely.

2. Inference Concerning Long Steady-State Delays in M/G/1-Like Systems

Consider a single service system approached by stationary Poisson (λ) traffic with λ known. Service times, or message lengths, X, are independently distributed as $F_X(x)$; assume $\lambda E[X] \equiv \rho < 1$. This is the M/G/1 system. Let observations of the service times be all that is known about F: denote these by $x_1, x_2, \ldots, x_n$. The objective is to supply estimates of long-run characteristics of the system: in particular, estimates, and error assessments thereof, of the probability of a long delay experienced by an arriving message.

2.1 Virtual Waiting Time Transform; The Long Tail.

It is well known that if W(t) is the virtual waiting time in the M/G/1 and $\rho < 1$, then the Laplace transform

$$E[e^{-sW}] = \lim_{t\to\infty} E[e^{-sW(t)}] = \frac{1-\rho}{1-\rho[\frac{1-E[e^{-sX}]}{sE[X]}]} = \frac{1-\rho}{1-\rho\tilde{A}(s)} \tag{2.1}$$

where A(s) is the Laplace-Stieltjes transform of a distribution. It can also be seen that in various interrupted and priority service systems, including those in which the server "takes vacations," that the above can be changed to

$$E[e^{-sW}] = \frac{1-\rho}{1-\rho\tilde{B}(s)} \cdot \tilde{C}(s) \tag{2.2}$$

where possibly $\tilde{B}(s)$ is the transform of a completion time (effective service time, i.e. X including durations of a random number of interruptions occurring therein), and $\tilde{C}(s)$ is the transform of an honest distribution that accounts for effects of busy period starts; see Gaver (1968).

If $\tilde{A}(s)$ (or $\tilde{B}(s)$) exists for $s > s_0$, $s_0 < 0$ then there will be a smallest real zero, $-s = \kappa > 0$, of the denominator of (2.1),((2.2)), which can be used to show that

$$P\{W > w\} \sim D(\kappa)e^{-\kappa w}, \quad w \to \infty \tag{2.3}$$

One way of establishing the above exponential tail property is to introduce

$$\psi(s) = \frac{1-E[e^{-sW}]}{s} = \int_0^\infty P\{W > w\}e^{-sw}dw$$

into (2.1) which leads to

$$\tilde{\psi}(s) = \rho[\frac{1-\tilde{A}(s)}{s}] + \rho\tilde{A}(s)\psi(s) \tag{2.4}$$

equivalent to the terminating renewal equation, see Feller (1966), p. 362,

$$\overline{F}_W(w) \equiv P\{W > w\} = \overline{H}(w) + \rho\int_0^w \overline{F}(w-x)A(dx) \tag{2.5}$$

Introduce $\overline{F}_W^{\#}(w) = \overline{F}_W(w)e^{\theta w}$, θ real and positive, into (2.5)

$$\overline{F}_W^{\#}(w) = \overline{H}^{\#}(w) + \int_0^w \overline{F}^{\#}(w-x)\rho e^{\theta x}A(dx) , \tag{2.6}$$

and choose $\theta = \kappa$ so that

$$\int_0^\infty \rho e^{\theta x}A(dx) = \int_0^\infty A^{\#}(dx) = 1 , \tag{2.7}$$

yielding a standard renewal equation for $\overline{F}^{\#}$, to which the key renewal theorem applies, yielding as $w \to \infty$

$$\lim_{w\to\infty} \overline{F}^{\#}_W(w) = \lim_{w\to\infty} \overline{F}_W(w)e^{\kappa w} = D(\kappa) = \frac{\int_0^\infty \overline{H}^{\#}(w)\,dw}{\int_0^\infty x A^{\#}(dx)} , \tag{2.8}$$

equivalent to (2.3). A similar expression is available for (2.2).

2.3 Statistical Estimation of Probability of Long Wait.

If data are available on service times (message lengths) then an estimate of the requisite transform is

$$\hat{E}[e^{-sX}] = \frac{1}{n}\sum_{i=1}^{n} e^{-sx_i} \tag{2.9}$$

and $\hat{\kappa}$ is the unique positive solution of this sample equivalent of (2.7):

$$\lambda\{\frac{1}{\theta}(\frac{1}{n}\sum_{i=1}^{n} e^{\theta x_i} - 1) = 1 . \tag{2.10}$$

A three-term Taylors' series expansion around $\theta = 0$ gives an initial estimate

$$\hat{\kappa} \simeq 2\left(\frac{1-\lambda\overline{x}}{\overline{x^2}}\right) \tag{2.11}$$

where $\overline{x^k} = (x_1^k + \ldots + x_n^k)/n$, the <u>kth</u> sample moment; (2.11) is recognized as being the non-parametric sample version of the exponential distribution parameter obtained by normalizing to $W/E[W]$ and letting $\rho \uparrow 1$ (the heavy traffic approximation). A search or Newton-Raphson iteration starting from (2.11) quickly yields $\hat{\kappa}$ as accurately as is necessary.

Some theoretical asymptotic results concerning modes of behavior of $\hat{\kappa}$ will now be sketched; more detail will be presented elsewhere. First, it is essential that $E[e^{\kappa X}] < \infty$ in order for (2.7) to hold, and so by continuity $\hat{f}^{(m)}(\kappa) = E[X^m e^{\kappa X}] < \infty$ for any integer m. Now express (2.10) as

$$\hat{f}(\infty) = \lambda\{\frac{1}{\theta}(\frac{1}{n}\sum_{i=1}^{n} e^{\theta x_i} - 1)\} - 1 \tag{2.12}$$

and expand in Taylor's series around κ: since $\hat{f}(\hat{\kappa}) = 0$, for some ϕ

$$0 = \hat{f}(\kappa) + (\hat{\kappa} - \kappa)\hat{f}'(\kappa) + \frac{1}{2}(\hat{\kappa} - \kappa)^2\hat{f}''(\phi\kappa) , \tag{2.13}$$

so

$$\hat{\kappa} - \kappa = \frac{\hat{f}(\kappa)}{\hat{f}'(\kappa) + \frac{1}{2}(\hat{\kappa}-\kappa)\hat{f}''(\phi\kappa)} \tag{2.14}$$

Note that if $E[e^{2\kappa X}] < \infty$ then $Var[e^{\kappa X}] < \infty$ and

$$Var[\hat{f}(\kappa)] = \frac{\lambda^2}{\kappa^2}\cdot\frac{1}{n}\, Var[e^{\kappa X}] , \tag{2.15}$$

while

$$E[\hat{f}(\kappa)] = 0 .$$

Additionally, $\hat{f}'(\kappa) \to E[\hat{f}'(\kappa)]$ by the weak law of large numbers and $\hat{\kappa}-\kappa \to 0$ in probability, so by the central limit theorem,

$$\sqrt{n}(\hat{\kappa}-\kappa) \cdot \frac{E[\hat{f}'(\kappa)]}{\sqrt{\frac{\lambda^2}{\kappa^2}\mathrm{Var}[e^{\kappa X}]}} \sim N(0,1) \tag{2.16}$$

the standard Normal/Gaussian distribution. If, however, $E[e^{2\kappa X}] = \infty$, then if any kind of limiting distribution is to exist, the components

$$e^{\kappa X_i}$$

of the sum $\hat{f}(\kappa)$ must be in the domain of attraction of a stable law, which will be assumed. In any case $\hat{f}'(\kappa) \to E[\hat{f}'(\kappa)]$ as before, so a proper normalization by a power of n shows that $(\hat{\kappa}-\kappa)$ has a stable law as limiting distribution. Fortunately, the relatively well-behaved Normal/Gauss limit prevails when traffic intensity is relatively high ($\rho > 1/2$ in case $X \sim \mathrm{Exp}(\mu)$, for example).

2.4 Assessing Variability of the Estimates.

Having obtained an estimate of the parameter κ, and of the probability of a waiting time exceeding large t, which involves κ, it is desirable to appraise the errors involved. Two error sources are: the systematic error (bias) resulting from fitting an incorrect model, and the effect of finite sample size (random error). The latter is the easiest to evaluate, provided the underlying model is nearly correct. The bias issue is more difficult to deal with; one approach is to begin by fitting an elaborate, multiparameter model and then to check for the contribution of extra parameters in a prediction context. Since this paper takes a non-parametric approach to estimation, the bias resulting from an incorrect specification of the service time distribution is presumably absent, although the assumptions of stationary Poisson arrivals, and independence are still reflected in the basic transform (2.1), and hence in the estimates. Unfortunately, classical procedures that, for example, avail themselves of the asymptotic approximations of properties of maximum likelihood parameter estimates to estimate sampling errors are no longer available in the present environment. We have chosen instead to investigate the performance of several modern procedures often recommended for obtaining standard errors of estimate and confidence intervals: the jackknife (Quenouille, Tukey, Miller, Hinkley, and others) and the bootstrap (Efron and associates and others). These methods, particularly the bootstrap, which involves simulation, are computer intensive, but are the only alternatives currently known for dealing with complex situations such as that at hand.

The Jackknife

The above name refers to a procedure originally introduced by Quenouille for bias reduction (1956), and adapted by Tukey (1958) to obtain approximate confidence intervals. Suppose interest is on a parameter θ (e.g. our κ, or some function thereof) that is estimated by θ, using a complex calculation from data $(x_1, x_2, \ldots, x_n)$, just as $\hat{\kappa}$ is. The idea is that of assessing variability by recomputing κ after removing independent subgroups of data of equal size, and then using the recomputed $\hat{\kappa}$ values to estimate a variance, which is in turn applied to state a standard error or a two-sided confidence interval that contains the true θ with specified confidence. A few details follow; for more, see Efron (1982) and his more recent work, or Mosteller and Tukey (1977). The actual calculation involves splitting n into g disjoint groups of size m; n = mg. Then calculate $\theta_{(-j)}$, j = 1,2,...,g: the estimate of θ that omits the jth group. Now Tukey (but not Efron) computes pseudo-values

$$y_j = g\hat{\theta} - (g-1)\hat{\theta}_{(-j)}$$

which are then treated as independent: use $\bar{y} = \hat{\theta}_{JK}$ or the point estimate of θ, and its approximate variance as

$$s_y^2/g = \sum_{j=1}^{g}(y_j-\bar{y})^2/(g-1)(g) = \sum_{j=1}^{g}(\hat{\theta}_{(-j)}-\bar{\hat{\theta}}_{(-j)})^2(g-1)/g,$$

as it turns out. Tukey recommends referring $\bar{y}$ to Student's t with g-1 degrees of freedom to obtain confidence limits. As the name "jackknife" is intended to imply, the tool is inexact and a bit crude for small samples--just as a true jackknife is not well-adapted for delicate surgery. It is a handy non-parametric option.

The Bootstrap

In simplest form the bootstrap suggests creating from observations (x_i, $i = 1,\ldots,n$) the empirical distribution

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(x_i,x); \qquad I(x_i,x) = \begin{cases} 1, & x_i \leq x \\ 0, & x_i > x \end{cases} .$$

Then one re-samples with replacement from F_n to obtain a bootstrap sample of size n, from which $\hat{\theta}$ (e.g. $\kappa(1)$) is calculated. This process is repeated B (e.g. 300-500) times, and the empirical distribution, $F_{\hat{\kappa}}$, of the resulting estimates, $\{\hat{\kappa}(b), b = 1,2,..,B\}$ is employed as an approximation to the sampling distribution of $\hat{\kappa}$: the upper and lower 10% points of $F_{\hat{\kappa}}$ approximate two-sided 80% confidence limits for $\hat{\kappa}$, for instance.

3. Transient Response: Exponential Characterizations

Applications of queueing theory frequently require assessment of service system transient behavior, particularly when the system is initially idle and heavy traffic conditions prevail, for then approach to steady-state conditions may be slow. Odoni and Roth (1983) have reviewed work on the nature of the approach to steady-state conditions, and have investigated characterizations of that approach that are simply stated and comprehensible, being exponential. In this section we elaborate upon the theme of exponential approach. Work by Abate and Whitt (1985) has similar aims, but proceeds somewhat differently.

3.1 From Transform to Exponential

Suppose $\{X(t), t \geq 0\}$ is the state variable of a process, and that expressions for

(a) $\tilde{\psi}_X(s) = \int_0^\infty e^{-st}E[\psi(X(t))|X(0)]dt = \int_0^\infty e^{-st}\psi_X(t)dt$ (3.1)

(b) $\lim_{t\to\infty} E[\psi(X(t)|X(0)] < \infty$

are known. An approximate representation of

(c) $\psi_X(t) = E[\psi(X(t))|X(0)]$

is desired. The proposed representation is a simple exponential interpolation of the form

$$\tilde{\psi}_X(t) = \psi_X(0)e^{-\beta t} + \psi_X(\infty)(1 - e^{-\beta t}) , \tag{3.2}$$

where β governs the rate of approach to the steady-state value. The proposal is

to identify β by minimizing the integrated squared error

$$\Delta(\beta) = \int_0^\infty \{\psi_X(t) - \psi_X(0)e^{-\beta t} + \psi_X(\infty)(1 - e^{-\beta t})\}^2 w(t)dt \ . \tag{3.3}$$

The weight function w(t) may be made to focus upon values of β appropriate for t-regions of interest. To optimize on β, study

$$\frac{d\Delta(\beta)}{d\beta} = K \int_0^\infty \{\psi_X(t) - \psi_X(0)e^{-\beta t} - \psi_X(\infty)(1 - e^{-\beta t})\} te^{-\beta t} w(t)dt \tag{3.4}$$

$$= 0 \ .$$

This is equivalent to

$$\int_0^\infty \psi_X(t)w(t)te^{-\beta t}dt = \psi_X(0)\int_0^\infty e^{-2\beta t} tw(t)dt + \psi_X(\infty)\int_0^\infty (1-e^{-\beta t})e^{-\beta t} tw(t)dt \tag{3.5}$$

If $w(t) \equiv 1$ the above can be expressed in terms of the derivative of the Laplace transform $\tilde{\psi}_X(s)$:

$$-\frac{d}{d\beta}\tilde{\psi}_X(\beta) = \frac{1}{\beta^2}[\frac{1}{4}\psi_X(0) + \frac{3}{4}\psi_X(\infty)] \ , \tag{3.6}$$

often a transcendental equation that must be solved numerically for β. Clearly, if no solution or multiple solutions exist then the simple universal form (3.2) is called into question, but in such cases weight functions are usefully invoked. If interest centers on ultimate approach ($t \to \infty$) to $\psi_X(\infty)$ then the smallest β satisfying (3.6) is of interest.

3.2 The M/G/1 Example.

As an important illustration of the above ideas, let $X(t) = W(t)$ be the virtual waiting time in an M/G/1 system, and consider $\psi_W(t) = E[W(t)|W(0)=0]$. We know that

$$\tilde{\psi}_W(s) = \frac{\rho-1}{s^2} + p_{oo}(s)\cdot\frac{1}{s} \tag{3.7}$$

where

$$p_{oo}(s) = \frac{1}{s+\lambda[1-b(s)]} = \frac{1}{s}\frac{1}{1+\lambda[\frac{1-b(s)}{s}]} = \frac{1}{s}h \tag{3.8}$$

b(s) being the transform of a busy period duration, satisfying

$$b(s) = \hat{F}_X(s + \lambda(1-b(s))) \ ;$$

$\hat{F}_X$ is the Laplace-Stieltjes transform of the service time or message length distribution F_X. Numerical solution of (3.6) is now possible. An approximation to the (smallest) value of β governing the ultimate approach ($t \to \infty$) when $\rho \uparrow 1$, i.e. in the heavy-traffic situation, is

$$\beta \simeq \sqrt{\frac{3}{4}\frac{\rho E[X^2]}{1-\rho}\{\frac{1}{-h'''(0)}\}} \ . \tag{3.9}$$

In general, $h'''(0)$ will involve higher moments of the service time; in case $X \sim Exp(\mu)$ considerable simplification occurs and

$$\beta \simeq \mu(1-\rho)^2\{8(1+\rho)\}^{-0.5} \tag{3.10}$$

which will not, of course agree exactly with formulas or numerical results given previously by other authors, e.g. see Odoni and Roth (1983) being a compromise ("Procrustean") interpolation of a single exponential over the infinite t-range. In order to focus upon a β-value relevant to ultimate approach it has been found that a weight procedure of the form $w_{k+1}(t) = [1-\exp(-\beta(k)t)]$ is workable and tractable; start with $\beta(0) = 0$ (the unweighted procedure described), then introduce w_{k+1} to determine β_{k+1} and iterate to convergence, which occurs rapidly. A similar approach should apply to find a β appropriate for small t.

The procedures described above should be applicable in the data-driven context of the problem of Section 1: a first step is to introduce the empirical transform $\hat{F}_X$ and from it proceed to $\hat{b}(s)$, to $\hat{\hat{\psi}}_W(s)$, and then to $\hat{\beta}$. Progress in this area, and details of the above investigations will be reported elsewhere.

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